



Zoran D. Perović

ACCURACY OF NUMERICAL METHODS FOR ASSESSMENT OF FATIGUE CRACK GROWTH IN WELDED JOINTS

TAČNOST NUMERIČKIH METODA ZA PROCJENU RASTA ZAMORNE PUKOTINE U ZAVARENIM SPOJEVIMA

Originalni naučni rad / Original scientific paper

UDK / UDC:

Rad primljen / Paper received:

Februar 2014.

Ključne riječi: određivanje faktora intenziteta napona, procjena zamornog vijeka, zavareni spoj

Rezime

Zamorna pukotina se može otkriti u raznim konstrukcijama, podvrgnutim cikličnom opterećenju, u toku inspekcije. U cilju da se predvidi preostali zamorni vijek, mogu se koristiti razne numeričke metode. Na samom početku treba odrediti faktor intenziteta napona (SIF) koji opisuje polje napona u blizini vrha pukotine. Postoji nekoliko metoda (i formula u nekim slučajevima) za određivanje faktora intenziteta napona. Zatim treba izabrati jednačinu brzine rasta zamorne pukotine i riješiti je pomoću neke numeričke metode. U ovom radu je razmotrena razlika konačnih rezultata ove procjene koja je posledica raznih primijenjenih metoda i formula u ovom postupku. Ova analiza je primijenjena na izabrani zavareni spoj.

1. INTRODUCTION

The big part of fatigue life is spent in fatigue crack propagation. Driving force of this process is stress intensity factor (SIF). Various numerical methods are developed in order to determine SIF. Because of its efficiency finite element method (FEM) is most frequently used e.g. virtual crack extension technique, the J-integral method, special elements with introduced $1/\sqrt{r}$ stress singularity by employing

$$K = M_k F \sigma \sqrt{\pi a} \quad (1)$$

$$F = \left[M_1 + M_2 \left(\frac{a}{t} \right)^2 + M_3 \left(\frac{a}{t} \right)^4 \right] f_\phi f_w g / Q \quad (2)$$

Adresa autora / Author's address:

Department of Mechanical Engineering, University of Montenegro George Washington bb, 81000 Podgorica Montenegro

Rad je u izvornom obliku objavljen u Zborniku sa savetovanja „ZAVARIVANJE 2014“ održanog na Borskom jezeru 4-7. Juna 2014, gde je i izlagan

Key words: SIF determination, fatigue life assessment, welded joints

Abstract

Fatigue crack could be detected in various structures, subjected to cyclic loading, during the inspection. In order to predict remaining fatigue life, various numerical methods can be used. At the very beginning, stress intensity factor (SIF) describing stress field in vicinity of the crack tip, should be determined. There a few methods (and formulas in some cases) for SIF determination. Then equation for crack growth rate should be chosen and solved by using one of the numerical methods. The difference in final results of the assessment as a consequence of different applied methods and formulas in this procedure for welded joints is considered in this paper.

a simple function, degenerated isoparametric elements etc.

2. NUMERICAL ANALYSIS

In this paper SIF is determined by using Raju & Newman solution for surface semielliptical crack in flat plate [1] subjected to the remote uniform tensile stress σ and Albrecht's solution for geometry correction factor for fillet welded joints [2]:



where:

$$M_1 = 1.13 - 0.09\left(\frac{a}{c}\right); \quad M_2 = -0.54 + \frac{0.89}{0.2 + \frac{a}{c}}; \quad M_3 = 0.5 - \frac{1.0}{0.65 + \left(\frac{a}{c}\right)} + 14\left(1 - \frac{a}{c}\right)^{24}$$

$$f_\varphi = \left[\left(\frac{a}{c}\right)^2 \cos^2 \varphi + \sin^2 \varphi \right]^{0.25}; \quad f_w = \left[\sec\left(\frac{\pi c}{w} \sqrt{\frac{a}{t}}\right) \right]^{0.5}; \quad g = 1 + \left[0.1 + 0.35\left(\frac{a}{t}\right)^2 \right] (1 - \sin \varphi)^2$$

$$Q = \left[1 + 1.464\left(\frac{a}{c}\right)^{1.65} \right]^{0.5}$$

where a, c = length of the minor and major semi-axis of the elliptical crack, respectively; t, w = thickness and width of the main plate of the welded joint, respectively; φ = angle that describes the

location at the crack front with respect to the major axis of the ellipse. The geometry correction factor M_k , is given by

$$M_k = \frac{2}{\pi} \sum_{i=1}^n \frac{\sigma_{b_i}}{\sigma} \left(\arcsin \frac{b_{i+1}}{a} - \arcsin \frac{b_i}{a} \right) \tag{3}$$

where σ = nominal stress in critical section of uncracked joint, σ_{b_i} = normal stress in a finite element between the distance b_i and b_{i+1} (Fig.1b). It accounts for the effect on K of a stress concentration produced by a structural detail. This

formula was obtained from Eq.4, the solution [3] for a central crack of length $2a$ in an infinite plate with two equal pairs of splitting forces P , applied at $x = \pm b$ (Fig.1a).

$$K = \frac{2P}{\sqrt{\pi a}} \frac{a}{\sqrt{a^2 - b^2}} \tag{4}$$

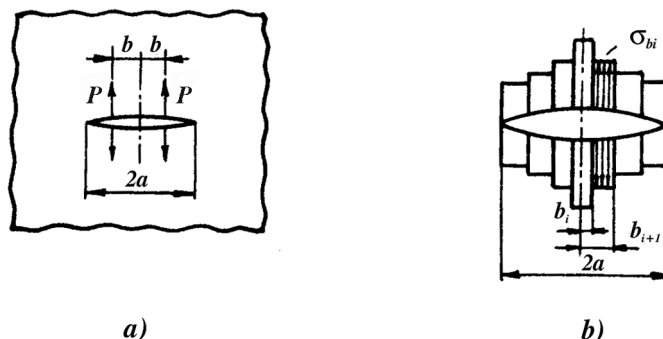


Fig.1. Crack in infinite plate subjected to: a) two pairs of equal splitting forces; b) pairs of discrete stresses



Verreman *et al.* [4] used this method for determination of the M_k factor of a cruciform-welded joint and compared it with the accurate solution obtained by Smith [5] using high-order crack tip elements with an inverse square root singularity. They reported differences smaller than 6%, so this method can be considered accurate for engineering

purposes. The advantage of Albrecht's method is that only one stress analysis needs to be made for each joint geometry, i.e. the stress analysis of an uncracked joint. The computer program was made based on Eq.(1-3) in order to determine SIF for welded joint shown in Fig.2.

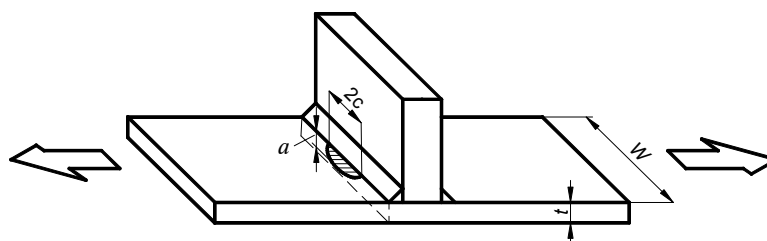


Fig.2. Welded joint ($t = 8 \text{ mm}$)

Calculated values of SIF for one arbitrarily chosen stress level are shown in Fig.3. These results are compared with those ones given in literature. The standard BS 7910:2005 [6] gives formula for SIF for cracks in welded joints, but it is valid for weld profile with sharp radii ($<0.1t$). The solution were obtained by curve fitting to individual finite element analyses [7]. One can see from ref. [7] that mentioned formula was obtained for $r/t = 0$ and recommended

for as-welded joints (r =weld toe radius). In ref. [7] was performed also second analysis for $r/t=0.1$ and obtained formula was recommended for welded joint with improved profile (TIG dressed or ground joints). That formula has the same shape like formulas (1) and (2) with magnification factor M_k (accounts for the effect on K of a stress concentration produced by a structural detail) given by:

$$M_k = f_1\left(\frac{a}{t}, \frac{a}{c}\right) f_2\left(\frac{a}{t}, \frac{L}{t}\right) f_3(\theta) \tag{5}$$

where

$$f_1\left(\frac{a}{t}, \frac{a}{c}\right) = A_1\left(\frac{a}{t}\right)^{A_2} + A_3\left(1 - \frac{a}{t}\right)^{A_4} + \left[A_5\left(\frac{a}{t}\right) + A_6\right]$$

$$f_2\left(\frac{a}{t}, \frac{L}{t}\right) = A_7\left(\frac{a}{t}\right)^{A_8} + A_9\left(1 - \frac{a}{t}\right)^{A_{10}}$$

$$A_1 = -3.2172(a/c)^2 + 8.9931(a/c) - 7.7356$$

$$A_2 = -0.22457(a/c)^2 - 0.41009(a/c) + 0.86071$$

$$A_3 = 0.65009(a/c)^2 - 0.76603(a/c) + 1.0351$$

$$A_4 = 0.10745(a/c)^2 - 11.039(a/c) + 30.557$$

$$A_5 = 1.2494(a/c)^2 - 7.1510(a/c) + 9.4916$$

$$A_6 = 0.33693(a/c)^2 + 0.23884(a/c) + 2.3341$$

$$A_7 = -0.0021981(L/t)^2 + 0.0066388(L/t) + 0.23244$$

$$A_8 = 0.098096(L/t)^2 - 0.22280(L/t) + 0.19344$$

$$A_9 = 0.015584(L/t)^2 + 0.026458(L/t) + 0.31065$$

$$A_{10} = -0.29651(L/t)^2 + 1.2995(L/t) + 1.0362$$

$$f_3(\theta) = 1.0$$

where L = attachment footprint width (thickness of attachment plus both welds).



3. RESULTS AND DISCUSION

Weld toe radius effect. These two methods (eq.1,2,3 and eq.1,2,5) are applied on welded joint (Fig. 2) in order to

determine SIF for surface semielliptical crack. Results of the calculation are shown and compared in Table. 1 and figure 3.

a, mm	K, MPa(m) ^{0.5}		K, MPa(m) ^{0.5}		K, MPa(m) ^{0.5}		K, MPa(m) ^{0.5} Eq.(1,2,5)
	r/t=0.1 Eq.(1,2,3)	Δ, %	r/t=0.2 Eq.(1,2,3)	Δ, %	r/t=0.4 Eq.(1,2,3)	Δ, %	
0.1	2.417	-1.4	2.240	-9.5	1.957	-25.3	2.452
0.2	2.910	-9.3	2.727	-16.6	2.523	-26.1	3.181
0.3	3.285	-10.4	3.122	-16.2	2.972	-22.0	3.627
0.5	3.818	-10.3	3.814	-10.5	3.669	-14.8	4.213
0.8	4.470	-8.5	4.546	-6.7	4.459	-8.8	4.850
1.2	5.243	-6.4	5.398	-3.4	5.343	-4.4	5.581
1.6	5.982	-4.6	6.168	-1.4	6.144	-1.8	6.255
1.9	6.524	-3.2	6.717	-0.3	6.718	-0.3	6.736
2.5	7.612	-0.7	7.808	+1.9	7.800	+1.7	7.666
3.2	8.882	+1.5	9.137	+4.4	9.091	+3.9	8.748
4.3	11.024	+4.2	11.348	+7.2	11.225	+6.1	10.581
5.9	14.221	+1.9	14.762	+5.8	14.496	+3.9	13.956
7.0	16.423	-4.0	16.849	-1.4	16.712	-2.2	17.078

Table 1. Comparison of SIF values obtained using Eq.(1,2,3) for various r values and those ones obtained by using Eq.(1,2,5)

Δ=Difference among SIF determined by Eq.(1,2,3) and that one determined by Eq.(1,2,5)

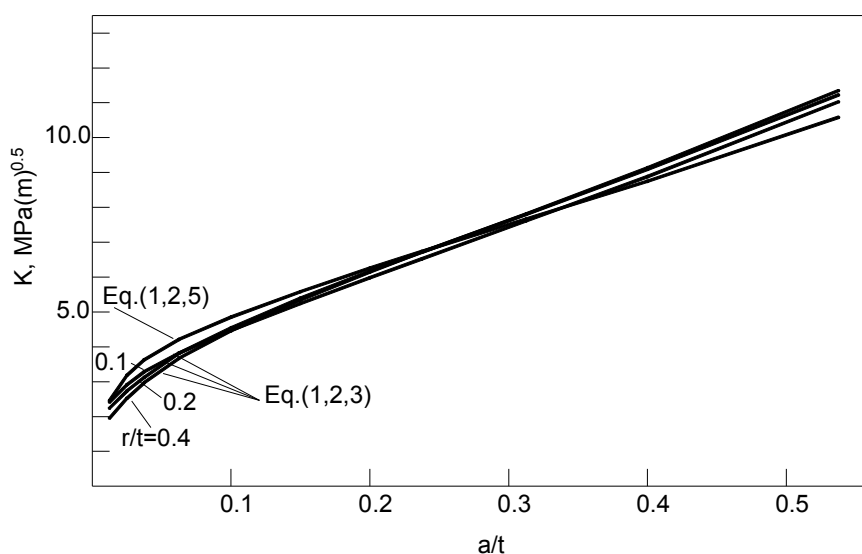


Fig.3. Stress intensity factor values obtained using Bowness and Albrecht solution (σ = 100 MPa)



Weld toe radius effect on SIF magnitude exist until the crack depth is $< 0.1t$ and differences determined in this work for this region are 10% for $r/t=0.1$, 16% for $r/t=0.2$ i 26% for $r/t=0.4$. It can be concluded that Eq.5 gives more conservative results for bigger weld toe radii in the vicinity of weld toe. Differences of SIF values, for bigger crack

depth, are smaller than 7% as a consequence of accuracy of these methods. It is interesting to determine how much these differences in SIF values affect the determination of the crack propagation life. The crack propagation life N_p is obtained from the equation:

$$N_p = \frac{1}{4.9 \times 10^{-12} (\Delta\sigma)^3} \int_{a_i}^{a_f} \frac{da}{[F(a)\sqrt{\pi a}]^3} \tag{6}$$

This equation was solved by the 32-point Gaussian quadrature method. The obtained results are

shown in Table 2. and Figure 4.

a_i/t	N_p		N_p		N_p		N_p
	$r/t=0.1$	$\Delta, \%$	$r/t=0.2$	$\Delta, \%$	$r/t=0.4$	$\Delta, \%$	
	Eq.(1,2,3,6)		Eq.(1,2,3)		Eq.(1,2,3)		Eq.(1,2,5,6)
0.05	1323845	+9.3	126072	+11.2	129425	+14.3	113336
0.1	80191	+3.3	79164	+2.0	79013	+1.8	77619
0.2	39516	-5.4	38705	-7.6	38218	-9.0	41644

Table 2. Comparison of crack propagation lives N_p obtained using Albrecht's method and Bowness method for various r values ($\Delta\sigma=300$ MPa)

Δ =Difference among N_p determined by Eq.(1,2,3,6) and that one determined by Eq.(1,2,5,6)

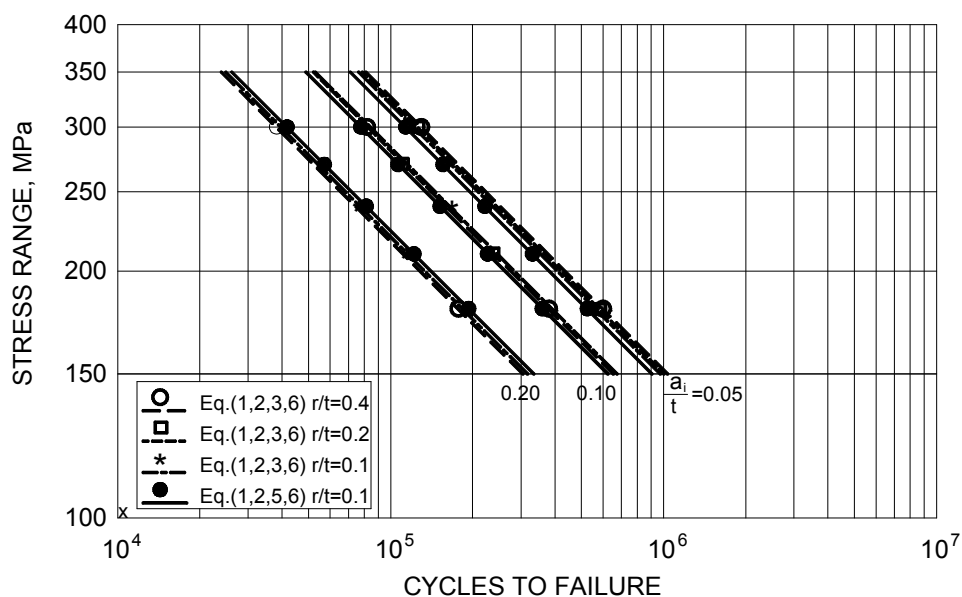


Fig.4 Effect of weld toe radius on fatigue life



Crack shape effect

Fatigue cracks emanating from stress concentrators (weld toe in this case) have semielliptical shape. Cracks change their shape

(aspect ratio $a/2c$) during their growth. At this place is presented analysis how much crack shape affect on stress intensity factor and crack propagation life. Results of this analysis is shown in Fig.5. and Fig.6.

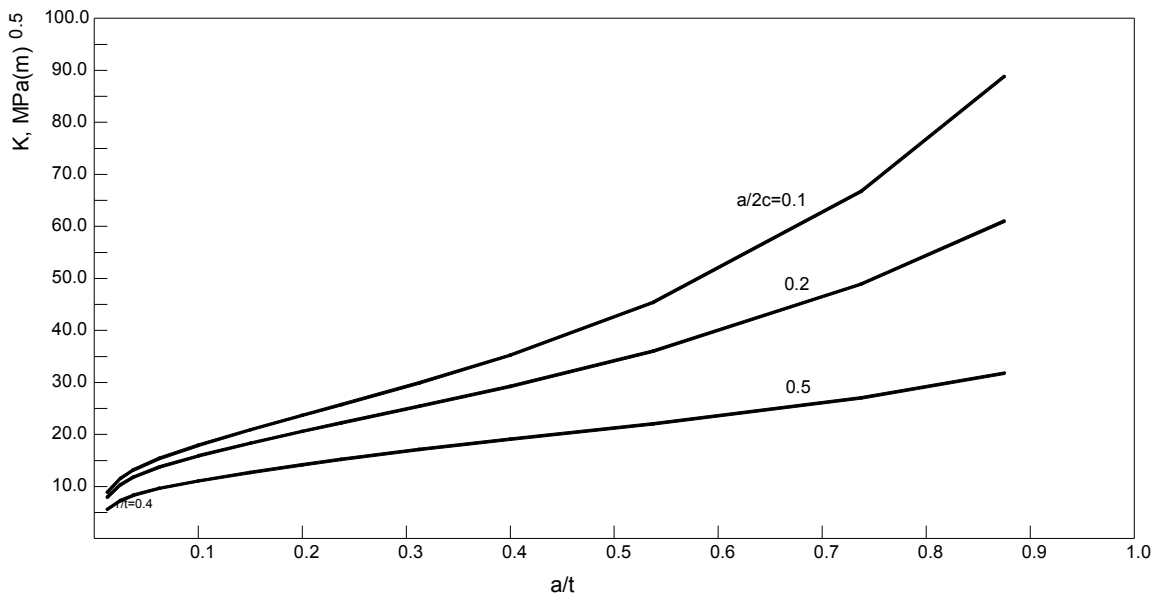


Fig.5 Crack shape effect on stress intensity factor; ($\sigma = 300 \text{ MPa}$)

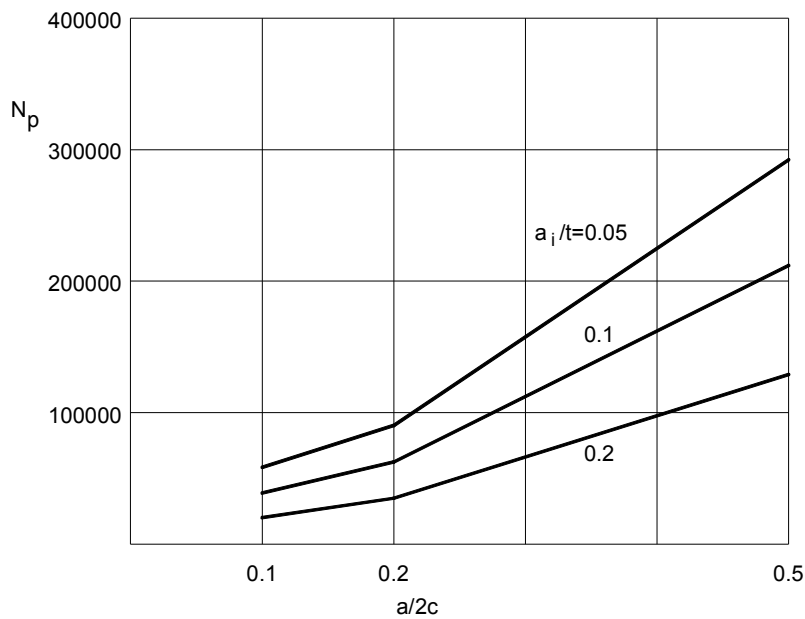


Fig.6 Crack shape effect on fatigue crack propagation life; ($\sigma = 300 \text{ MPa}$)



4. CONCLUSIONS

Accuracy of various methods for SIF determination is considered in this paper. Stress intensity factor for T-welded joint is obtained by using two methods: Albrecht's method and Bowness formula. The results show good agreement-difference is smaller than 7% for relative crack depth bigger than $a/t = 0.1$. The difference in corresponding crack propagation lives is from +3% to -9%. For cracks smaller than 0.1t difference increases with increase of the weld toe radius: 10% for $r/t=0.1$, 16% for $r/t=0.2$ and 26% for $r/t=0.4$. The difference in corresponding crack propagation lives is from +9% to 14%. Bowness formula gives conservative solution for TIG dressed and ground welded joints. The influence of crack shape on SIF and crack propagation life is considered as well. For instance decrease of aspect ratio $a/2c$ from 0.5 to 0.1 increases SIF 59% for $a/t=0.0125$ and 179% for $a/t=0.875$. Corresponding crack propagation lives decrease 5 times (at $\Delta\sigma=300$ MPa, $a/t=0.05$, $a/2c$ decreases from 0.5 to 0.1) and 6.4 times for (at $\Delta\sigma=300$ MPa, $a/t=0.2$, $a/2c$ decreases from 0.5 to 0.1). These examples illustrate big influence of crack shape on stress intensity factors and crack propagation lives.

REFERENCES

- [1] Raju, I. S., Newman, J. C., Stress Intensity Factors for a Wide Range of Semi-elliptical Surface Cracks in Finite Thickness Plates, *Engineering Fracture Mechanics*, **11** (4), 817-829 (1979)
- [2] Albrecht, P., Yamada, K., Rapid Calculation of Stress Intensity Factors, *Journal of the Structural Division*, **103** (2), 377-389, (1977)
- [3] Tada, H., Paris, P., Irwin, G. R., *The Stress Analysis of Cracks Handbook*, Del Research Corporation, Hellertown, Pa., 1973
- [4] Verreman, Y., Bailon J. P., Masounave, J., Fatigue Life Prediction of Welded Joints –a re-assessment, *Fatigue and Fracture of Engineering Materials and Structures*, **10** (1), 17-36, (1987)
- [5] Smith, I. J., The Effect of Geometry Changes upon the Predicted Fatigue Strength of Welded Joints, *Proceedings, Third International Conference on Numerical Methods in Fracture Mechanics*, Swansea, UK, 561-574, 1984
- [6] BS 7910. Guide to Methods for Assessing the Acceptability of Flaws in Metallic Structures, British Standards Institution, 2005
- [7] Bowness, D., Lee, M. M. K., Prediction of Weld Toe Magnification Factors for Semi-elliptical Cracks in T-butt joints, *International Journal of Fatigue*, **22** (5), 369-387, (200)