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DIRECT MEASUREMENT OF J INTEGRAL – A RETROSPECTIVE

DIREKTNO MERENJE J INTEGRALA – RETROSPEKTIVA

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Abstract

Direct measurement of J integral has been presented as a retrospective of more than 40 years of experience. Early efforts and experiments on welded wide plates and pressure vessels are presented and analysed in respect to the effect of material heterogeneity. Physical meaning of the J integral values obtained by direct measurements on a pressure vessel is explained to clarify its application. Different boundary conditions are also considered, to investigate options for simplified measurements. The possibility of applying simplified direct measurement of J integral for pressure vessel integrity assessment has been considered. Finally, it is noted that the measurement of strains by digital image correlation instead of strain gauges can make a new life of this technique.

Rezime

Direktno merenje J integrala predstavljeno je kao retrospektiva iskustva dužeg od 40 godina. Predstavljene su i analizirani rani naponi i eksperimenti na zavarenim širokim pločama i posudama pod pritiskom, u odnosu na uticaj heterogenosti materijala. Objašnjeno je fizičko značenje vrednosti J integrala dobijenih direktnim merenjima na posudi pod pritiskom kako bi se razjasnila njegova primena. Razmatrani su i različiti granični uslovi, kako bi se istražile opcije za pojednostavljena merenja, kao i mogućnost primene pojednostavljenog direktnog merenja J integrala za procenu integriteta posuda pod pritiskom. Na kraju, primećeno je da merenje deformacija pomoću digitalne korelacije slike umesto mernih traka može udahnuti novi život ovoj tehnici.



PROLOGUE – the role of Prof. Dr. Stojan Sedmak

Welding is an exceptional discipline because it is both a "hard" science and sophisticated skill. Serious scientists deal with the first aspect, and experienced welders are irreplaceable for the second, and together they form a team that guarantees the integrity of welded joints and structures. However, there always remains the problem of cracks as an inevitable nightmare of more complex materials and geometries. In other words, fracture mechanics of welded joints is always actual topic, the very mention of which immediately brings to mind the great man of the Serbian and Yugoslav welding scene - Stojan Sedmak. Therefore, this paper is dedicated to him, as one of the five legends of fracture mechanics. Let us briefly remember other four: Mladen Berković (computational mechanics), Aleksandar Radović (welding), Tihoslav Tošić, whose "YES, we will build Reversible Hydro Power Plant Bajina Bašta", when he was the general manager of GOSA, is directly and to the greatest extent responsible for the development of fracture mechanics of welded joints in our country. Finally, Branko Žeželj, who pushed the limits of building with prestressed concrete, so that the dome of hall 1 of the Belgrade Fair is still the "world champion" in the range of freely supported construction, according to Guinness records.

This paper is a retrospective of one significant topic which was in Stojan's focus for decades, and can serve at the same time as an excellent example to illustrate gigantic efforts of the man who positioned Serbia in a high place in Europe and the world in the field of fracture mechanics of welded joints. As proof of that claim, let's cite just a few facts:

- Stojan's successors are the President of European Structural Integrity Society (ESIS), and has been one of directors of the International Institute of Welding (the only one ever from exYu) until recently.
- The ESIS TC18 is dedicated to the memory of Prof. Stojan Sedmak, "who strongly contributed to ESIS", and delivers the award every two years to scientists that have strongly contributed to the structural integrity of welded structures.
- The 22nd European Conference on Fracture, was organized in Belgrade in 2018 with over 500 participants.

- This November, the first European-Chinese conference on structural integrity will be held in Belgrade.
- Journal "Structural Integrity and Life", established by him in 2001, has reached M22 rank in Ministry of Science classification.

In the text that follows, the works related to the direct measurement of the J integral will be briefly presented, with an emphasis on the application to pressure vessels, as an original contribution of our researchers under the leadership of Stojan Sedmak. This overview will be followed by some new interpretations of the old results.

1. Introduction

Direct measurement of J integral was introduced by Dave Read [1] back in eighties as a new option to evaluate J integral on wide plates, including welded joints. Main idea was to express J integral by strains, which were measured along wide plate outer paths, and by CMOD, which was measured at the same time during tensile loading. Namely, starting from the definition of J integral

$$J = \int_{\Gamma} W dy - \vec{T} \frac{\partial \vec{u}}{\partial x} ds \quad (1)$$

where x and y represent Cartesian coordinates with an origin at the tip of an edge crack, Fig. 1, W is the strain work density, \vec{T} is the tension vector on contour Γ (ABCD in Fig. 1), \vec{u} is the displacement vector. The stress-strain curve representing the elastic-ideally plastic material behaviour is given in Fig. 2. Two integrand expressions in Eq. (1), i.e. the strain energy member Wdy and tensile-bending member $\vec{T} \frac{\partial \vec{u}}{\partial x} ds$ are calculated for contour segments AB, CD and BC as follows.

The strain work density $W = \int \sigma_{ij} d\varepsilon_{ij}$, where σ_{ij} is the stress tensor and ε_{ij} is the strain tensor. For the plane stress state one gets: $\sigma_{zz} = 0$, $\sigma_{yz} = 0$, $\sigma_{zx} = 0$, hence $W = \int \sigma_{xx} d\varepsilon_{xx} + \sigma_{yy} d\varepsilon_{yy} + \sigma_{xy} d\varepsilon_{xy}$ i.e., for contour segments AB and CD, $W = \int \sigma_{yy} d\varepsilon_{yy}$, since stress tensor components along the free surface are all equal to zero, except σ_{yy} .

Tensile force \vec{T} equals zero along the contour segments AB and CD, since they are located at free surfaces, hence the second member in the J integral expression equals zero for these segments.



On segment BC, $dy = 0$, since this segment is parallel to the x axis, hence the first member equals zero for this segment. Tensile force \vec{T} is calculated as: $T_i = \sigma_{ij}n_j$ where n_j is the unit vector along the outer normal to the contour Γ . Taking plane stress state into account: $T_x = \sigma_{xx}n_x + \sigma_{xy}n_y$; $T_y = \sigma_{yx}n_x + \sigma_{yy}n_y$. Along segment BC, $n_y = 1$, and $n_x = 0$, so: $T_x = \sigma_{xy}$, $T_y = \sigma_{yy}$.

Shear component σ_{xy} can be neglected since the chosen contour segment is parallel to the x axis and at a sufficient distance from the crack. Component u_x of the displacement vector \vec{u} along segment BC can also be neglected, since there is no displacement in that direction. Hence the first integrand member in J integral for the contour segment BC is reduced to the product of the tensile force \vec{T} and the change in displacement vector \vec{u} along the x axis. Tensile component T_y is obtained from strain, measured using strain gauges at points B and C, Fig. 1, since all stress components except σ_{yy} can be neglected along the segment BC. Bending member $\partial u_y / \partial x$ along segment BC can be expressed as:

$$\frac{\partial u_y}{\partial x} = \frac{u_y(C) - u_y(B)}{x(C) - x(B)} \quad (2)$$

where displacements $u_y(C)$ and $u_y(B)$ at points C and B are measured using a linear variable differential transformer (LVDT). Variables $x(C)$ and

$x(B)$ represent coordinates at points C and B, and the difference between them, $x(C) - x(B)$, represents the specimen thickness. Strain ϵ_{yy} is obtained from strain gauges, which are set according to Fig. 3. These values of strain for calculating the work density W are limited to a finite number of locations along segments AB and CD, hence the value of the J integral is an approximation.

Calculation of tensile-bending member, J_{BC} , is reduced to multiplying $T_y (\approx \sigma_{yy})$ with $u(C) - u(B)$, since $ds = dx$. By using numerical integration, it is possible to determine the values of displacements $u_y(C)$ and $u_y(B)$ via strain distribution, using the following expressions:

$$u_y(C) = \int_D^C \epsilon_{yy} dy \quad \text{and} \quad u_y(B) = \int_A^B \epsilon_{yy} dy + \frac{CMOD}{2}$$

where CMOD is the crack mouth opening displacement, measured using a special gauge, Fig. 1b.

Direct numerical integration is used for calculating members in Eq.(1), since segments DC and AB are divided into smaller segments, each containing a single strain gauge. Each segment contribution is taken into account as the product of strain energy density, calculated from the measured strain from Eq. (2), and segment length. Thus, members J_{AB} , J_{BC} and J_{CD} are then added in the following way: $J = 2(J_{AB} + J_{CD} + J_{BC})$

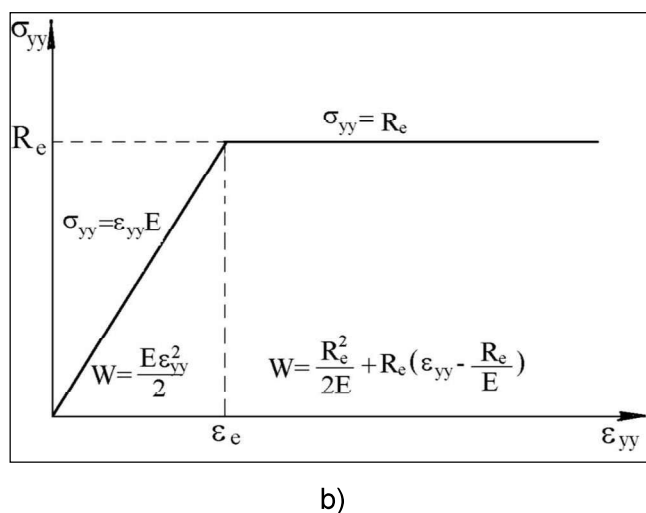
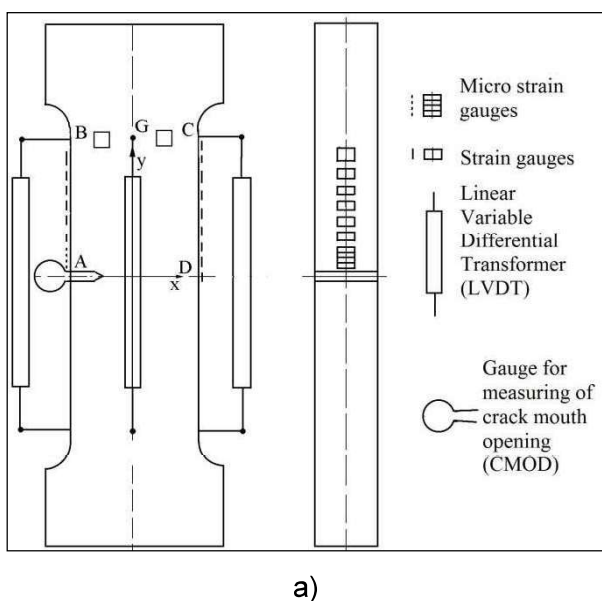


Figure 1. a) Instrumentation for tension tests of the edge cracked specimen, b) stress-strain curve, [2]

Slika 1. a) Oprema za ispitivanje zatezanjem uzorka sa ivičnim prslinama, b) kriva napon - deformacija, [2]



2. Literature overview

To start with, wide plates made with mismatched welded joints were tested to prove fitness-for-purpose of the large penstock in reversible HPP BB, as described in details in [3-5]. Basic concept was to compare J-R curves, obtained by direct measurement of J integral, with crack driving forces (CDF), obtained numerically for different crack lengths and pressure levels in a cylindrical pipe. To verify numerical model for CDF, the special technique was used to measure J integral directly on a cylindrical pressure vessel, as described in [6,7]. This issue is the main focus of this paper, as well.

Later on, direct measurement was used to test wide plates with undermatched and overmatched welded joints, as described in [2,8]. Toward this aim, J integral components SW and St were presented separately, to follow more precisely mismatching effects. Also, the effect of boundary conditions was analysed in [2,8] and will be discussed here as well.

Direct measurement of J integral, as a technique for material testing, was also presented at the International Conference on Computerization in Welding Info IV, [9], once it was recognized also as data collection and management process.

Both wide plates and cylindrical pressure vessel, were used in another experiment, performed in Skopje in late eighties, as a part of doctoral thesis of Todor Adziev, [10,11]. Focus of this experiment was on the effect of welding residual stresses and geometry imperfections on crack resistance. These effects were shown to be stronger than expected.

Yet another set of experiments with welded wide plates was performed afterwards in Skopje in the scope of D.Sc. thesis of Gorgi Adziev, with the main focus to assess structural integrity of a large spherical tank made of HSLA steel, [12,13]. Directly measured J integral values were used as J-R curves to find the critical point on corresponding CDF and thus the pressure which would lead to failure after elastic-plastic deformation of a welded joint.

In the meantime, similar experiment with pressure vessel was performed in Belgrade, in the scope of D.Sc. thesis of Mohamed Essamei, [14,15] with the focus on residual stresses. At the same time, detailed theoretical analysis of the method itself was performed to evaluate the effect of biaxial stress state and strengthening of material in plasticity, [2,16,17]. It was shown that these

effects are not important in the case of cylindrical vessels, but would be of utmost importance in the case of spherical vessel. The effect of material strengthening was not significant, as well, [2].

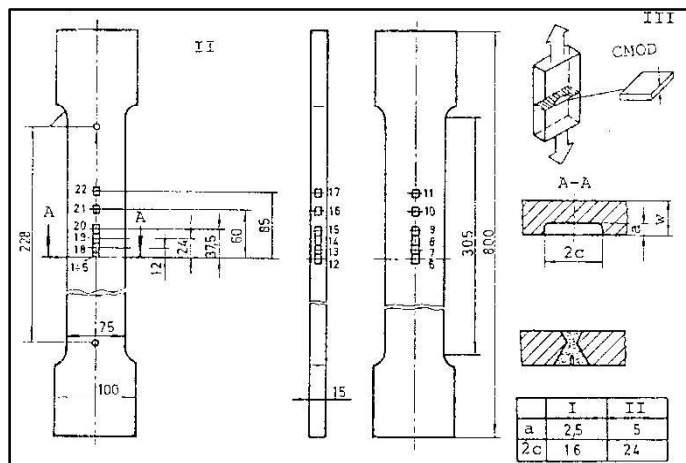
3. Effect of welded joint matching on crack behavior in welded joints

Heterogeneity of tensile properties has an important role in the behavior of welded joints, particularly if stress concentration caused by geometrical imperfections, such as angular distortion or misalignment, produce local plastic strains. To assess this effect tensile panels with surface cracks were made of Sumiten steel SM 80P and SM60 for three different matching levels of welded joints made by SAW:

1. Overmatching (SM 60 steel + US 80B wire);
2. Undermatching (SM 80P steel + US 49 wire);
3. Normal matching (SM 80P steel + US 80B wire), designed also as an undermatched joint.

These three matching levels were designed in accordance of the usual welding practice for steels above 700 MPa yield strength, which recommends undermatched welded joints to avoid problem with cold cracking.

In addition to tensile panels, three-point bend (3PB) specimens were made with cracks in parent metal (PM) and weld metal (WM), Fig. 2, so that non-standard specimen can be compared with the standard ones.





Experimental pressure vessel (Fig. 4.I), made of SM80P steel 16 mm thick plates, with fatigue pre-crack, positioned in a sample of SAW weld metal, was used for J integral direct measurement (Fig. 4.II). Fatigue pre-crack in WM center was produced in the segment (180x380 mm), cut out and re-welded to the pressure vessel. The instrumentation

for direct measurement of J integral around selected contour DCBAB'C'D", Fig. 4.III, consisted of strain gauges, Fig. 4.IV, and CMOD clip gauge. During the experiment crack grew into length, from initial value $2c = 64.25$ mm to the final value $2c = 80$ mm, with depth remaining constant, $a = 11$ mm, Fig. 4.II.

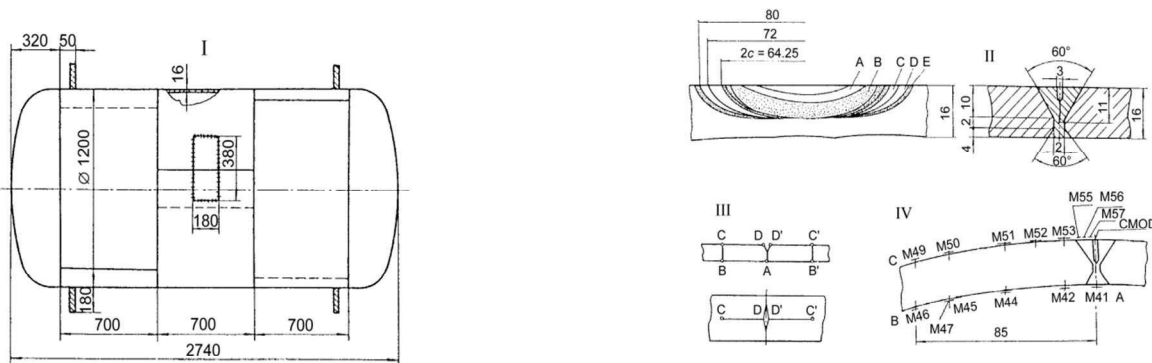


Figure 4. Experimental pressure vessel: I Shape and dimensions. II Details of the crack (A notch; B fatigue pre-crack; C, D stable crack growth during testing; E final fatigue crack). III Integration contour. IV Distribution of strain gauges

Slika 4. Eksperimentalna posuda pod pritiskom: I Oblik i dimenzije. II Detalji prsline (A zarez; B zamorna pre-prslina; C, D stabilan rast prsline tokom testiranja; E konačna zamorna prslina). III Kontura integracije. IV Raspodela mernih traka

For the crack length $2c = 64.25$ mm, vessel diameter $2R = 1184$ mm, wall thickness $W = 16$ mm, and Poisson's ratio $\nu = 0.3$, the shell parameter λ used for definition of CDF values for investigated pressure vessel, is:

$$\lambda = \left[12(1-\nu^2) \right]^{1/4} \frac{c}{\sqrt{R \cdot W}} = \left[12(1-0.3^2) \right]^{1/4} \frac{32.125}{\sqrt{1184 \cdot 0.016}} = 0.6$$

Crack driving force for an axial surface crack have been calculated by REI model, [18], for $\lambda = 0.6$, expressed in non-dimensional form ($\sqrt{J^*} = \sqrt{J \cdot E / 4aR_{p0.2}}$) and plotted as set of lines in Fig. 5, for different crack ratios a/W and different normalized pressures $p \cdot R / W \cdot R_{p0.2}$. Thereby, $R_{p0.2}$ is

taken as 585 MPa, [19]. One should notice that CDF values are obtained for the fixed values of crack length, $2c = 64.25$ mm, and depth, $a = 11$ mm. At the same time, using J integral direct measurement, it was possible to obtain the point "A" at the pressure 100 bar, expressed in the same non-dimensional form ($\sqrt{J^*} = 0.95$), Fig. 5. Corresponding value of CDF, calculated by REI model, is lower 33% (0.63). This difference can be attributed to the fact that crack has grown into length, making this comparison questionable, since REI model is valid for fixed crack dimensions. Taking crack growth into account, this difference is reasonable.

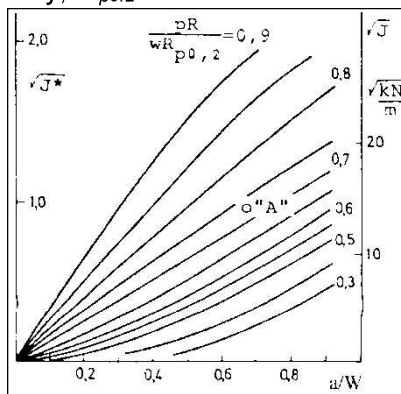


Figure 5. Crack Driving Forces vs. crack depth

Slika 5. Zavisnost sila rasta prsline od dubine prsline



4. Discussion

There are two questions that remained unanswered after all these experiments with direct measurements of J integral. The first one is the physical meaning of point "A" (Fig. 5) and the second one the practical use. Here, we will try to provide the answers.

Regrading point A, as already mentioned, the idea was to verify REI model for CDF calculation, which is mathematical fiction since the curves representing CDF are just a collection of points obtained for fixed crack dimensions and different pressure levels. Namely, if one follows any of CDF curves for the fixed pressure, they are just the collection of points for different crack length and by no means dependence of CDF on crack length during its growth. Therefore, crack growth during pressurizing up to 100 bar makes comparison of point A with such a mathematical fiction questionable. On the other hand, if one gets all sets of points representing J integral vs. crack length, that is practically J-R curve. If it is done with standard specimen, such a curve represents material resistance to crack growth and can be used in combination with CDFs to evaluate critical pressure values, as done and shown in number of references [xx]. Although J integral values were not recorded and neither was the crack length, it is obvious that point A is just one point in non-recorded J-R curve. But than again, let us emphasize that point A and non-recorded J-R curve would not be a measure of material crack resistance, but rather a measure of pressure vessel crack resistance. In other words, if J integral is directly measured on a full-scale pressure vessel, then its critical value simply corresponds to the leakage pressure (if dominant crack growth is into depth) or the pressure when catastrophic failure occurs (dominant crack growth into length). No need for CDFs and material J-R curves in that case.

To answer the second question, i.e. the practical use of direct measurement of J integral, the way is to start from the fact that its complexity prevented its applicability. Also, knowing strict regulations in respect to pressure vessel exploitation, with crack-

like defects and unacceptable, it is not likely that such a problem occurs in practice. Yet, real life is often intriguing. In this experience of these authors, there were a couple of issues with crack-like defects in pressure vessel welded joints which could not be repaired and replacement of the vessel was hardly an option. Fortunately, conservative, linear elastic fracture mechanics analysis proved that the vessel integrity was not jeopardized. If that was not the case, elastic-plastic fracture mechanics analysis, based on J integral would have been another option. Well, certainly not in the way as described here., leading us to try to find an alternative. And that is how the boundary conditions became an issue.

To elaborate this issue let us quote part of paper [2], reflecting Fixed point- and four-point bending – modification of boundary conditions: "If the ends of the plate are in the jaws of the testing machine so that they cannot rotate, then the boundary conditions change, or the tension-bending member becomes zero". Just to mention - in the original Read's paper there was a pin, [1]. This simple fact has not been noticed in the works dealing with such investigations, e.g. [20]. The situation is similar for pressure vessels, but in that case there is still a possibility for (small) rotation of loaded ends. Nevertheless, as a reasonable approximation, one can neglect such a small rotation. In that case, the J integral reduces to a difference of deformation energies at the smooth side and at the side of the crack, and CMOD measurement is not necessary. In practical terms, this means that all one needs to know is strain distribution on the outer side of a PV since the crack-like defects are typically on the inner side or somewhere along thickness, while strain energy from the inner side can be estimated even analytically, not to mention the finite element method.

From practical point of view, one should also keep in mind use of DIC for strain measurement instead of SGs, including the standard proof testing of a pressure vessel suspected to presence of crack-like defects from the inner side or along wall thickness.



5. Conclusions

Base on presented results and discussion following can be concluded:

- Direct measurement of J integral is powerful tool to assess material crack resistance and even the structural integrity of a component like pressure vessel
- This technique can be simplified in the case of clamped specimens and pressure vessels due to boundary conditions and may be reduced just to a measurement of strain distribution in the area opposite from the crack-like defect
- Using of DIC instead of SGs could mean new life for this simple technique.

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5. Zaključci

Na osnovu prikazanih rezultata i diskusije, mogu se izvući sledeći zaključci:

- Direktno merenje J integrala je moćan alat za procenu otpornosti materijala na širenje prslina, pa čak i za procenu strukturalnog integriteta komponente kao što je posuda pod pritiskom.
- Ova tehnika se može pojednostaviti u slučaju uklještenih epruveta i posuda pod pritiskom zbog graničnih uslova, a može se svesti samo na merenje raspodele deformacija u oblasti nasuprot defektu nalik prslini.
- Upotreba DIC (digitalne korelacije slike) umesto SG (mernih traka) može udahnuti novi život ovoj jednostavnoj tehnici.

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VESTI



NEWS

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Ova konferencija se smatra vodećom u oblasti zavarivanja cevovoda. Konferencija će se fokusirati na ključne komponente koje čine okosnicu globalnog energetskog sektora: cevovode, rezervoare i skladišta. Učesnici će imati priliku da istraže i analiziraju teme koje oblikuju budućnost industrije - od zastarelih praksi do revolucionarnih rešenja. Konferencija će okupiti širok spektar stručnjaka, uključujući inženjere, građevinske stručnjake, inspektore i stručnjake za održavanje. Fokus će biti na napretku, izazovima i inovacijama koje doprinose poboljšanju transporta, skladištenja i rada postrojenja.

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